Chapter 13 Notes Surface Areas and Volumes

Exercise 13.3

Q.5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Sol. For the circular cylinder:

Diameter = 12 cm Þ Radius = $\frac{12}{2}$ = 6 cm

Height (h) = 15 cm

$\therefore$ Volume = $\pi r^2 h$

$\Rightarrow$ Volume of total ice cream = $\frac{22}{7} \times 6 \times 6 \times 15 \text{ cm}^3$

For conical + hemispherical ice-cream cone:

Diameter = 6 cm $\Rightarrow$ radius (R) = 3 cm

Height of conc. (H) = 12 cm

Volume = (Volume of the conical part) + (Volume of the hemispherical part)

$$= \frac{1}{3}\pi R^2 H + \frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 [H + 2R]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 [12 + 2 \times 3] \text{ cm}^3 = \frac{22 \times 3}{7} \times 18 \text{ cm}^3$$

Let number of ice-cream cones required to fill the total ice cream = n.

$$\therefore n\left[\frac{22 \times 3}{7} \times 18\right] = \frac{22}{7} \times 6 \times 6 \times 15$$

$$\Rightarrow n = \frac{22}{7} \times 6 \times 6 \times 15 \times \frac{7}{22} \times \frac{1}{3} \times \frac{1}{18}$$

$$\Rightarrow n = 2 \times 5 = 10$$

Thus, the required number of cones is 10.

Q.6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cn × 10 cm × 3.5 cm?

Sol. For a circular coin:

Diameter = 1.75 cm

$\Rightarrow$ Radius (r) = $\frac{175}{200}$ cm

Thickness (h) = 2 mm = $\frac{2}{10}$ cm

$\therefore$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}$ cm^3 [$\because$ A coin is like a cylinder]

For a cuboid:

Length (l) = 10 cm, Breadth (b) = 5.5 cm and Height (h) = 3.5 cm

$\therefore$ Volume = $10 \times \frac{55}{10} \times \frac{35}{10}$ cm^3

Number of coins

Let the number of coins need to melt be 'n'

$$\therefore n = \left[10 \times \frac{55}{10} \times \frac{35}{10}\right] \div \left[\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}\right]$$

$$= 10 \times \frac{55}{10} \times \frac{35}{10} \times \frac{7}{22} \times \frac{200}{175} \times \frac{200}{175} \times \frac{10}{2} = 16 \times 25 = 400$$

Thus, the required number of coins = 400.

Q.7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Sol. For the cylindrical bucket:

Radius (r) = 18 cm

Height (h) = 32 cm

Volume = $\pi r^2 h$

$= \frac{22}{7} \times (18)^2 \times 32$ cm^3

$\Rightarrow$ Volume of the sand = $\left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm^3

For the conical heap:

Height (H) = 24 cm

Let radius of the base be (R).

$\therefore$ Volume of conical heap = $\frac{1}{3}\pi R^2 H = \left[\frac{1}{3} \times \frac{22}{7} \times R^2 \times 24\right]$ cm^3

Radius of the conical heap of sand:

$\because$ Volume of the conical heap of sand = Volume of the sand

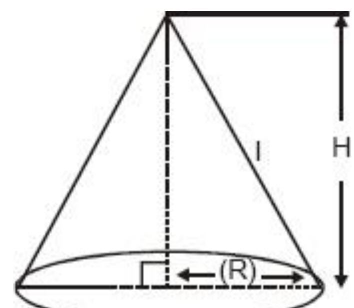
$$\therefore \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24 = \frac{27}{7} \times 18 \times 18 \times 32$$

$$\Rightarrow R^2 = \frac{22}{7} \times 18 \times 18 \times 32 \times 3 \times \frac{7}{22} \times \frac{1}{24} = 18 \times 18 \times 4 = 18^2 \times 2^2$$

$$\Rightarrow R = \sqrt{18^2 \times 2^2} = 18 \times 2 \text{ cm} = 36 \text{ cm}$$

Slant Height

Let 'l' be the slant height of the conical heap of the sand.



$\therefore \quad l^2 = R^2 + H^2$

$\Rightarrow \quad l^2 = 24^2 + 36^2$

$\Rightarrow \quad l^2 = (12 \times 2)^2 + (12 \times 3)^2$

$\Rightarrow \quad l^2 = 12^2 [2^2 + 3^2]$

$\Rightarrow \quad l^2 = 12^2 \times 13$

$\Rightarrow \quad l = \sqrt{12^2 \times 13} = 12 \times \sqrt{13}$

Thus, the required height = 36 cm and slant height = $12\sqrt{13}$ cm.

Q.8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column in 30 minutes (i.e., $\frac{1}{2}$ hr)

$$= \frac{10}{2} \text{km} = \frac{10000}{2} \text{m} = 5000 \text{ m}$$

$\therefore$ Volume of water flown in $\frac{1}{2}$hr

$$= 6 \times 1.5 \times 5000 \text{ m}^3 = 6 \times \frac{15}{10} \times 5000 \text{ m}^3 = 45000 \text{ m}^3$$

Since the above amount (volume) of water is spread in the form of a cuboid of height as 8 cm

$\left(=\frac{8}{100}m\right)$.

Let the area of the cuboid = a

$\therefore$ Volume of the cuboid = Area × Height = a × $\frac{8}{100}m^3$

Thus, a × $\frac{8}{100}$ = 45000

$\Rightarrow$ a = $\frac{45000 \times 100}{8} = \frac{4500000}{8}m^2$

$\Rightarrow$ a = 562500 m^2 = $\frac{562500}{10000}$ hectares = 56.25 hectares

Thus, the required area = 56.25 hectares.

Q.9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol. Diameter of the pipe = 20 cm

$\Rightarrow$ Radius of the pipe (r) = $\frac{20}{2}$ cm = 10 cm

Since, the water flows through the pipe at 3 km/hr.

$\therefore$ Length of water column per hour

(h) = 3 km = 3 × 1000 m = 3000 × 100 cm = 300000 cm.

$\therefore$ Volume of water = $\pi r^2 h$ = π × 102 × 300000 cm^3 = π × 30000000 cm^3

Now, for the cylindrical tank,

Diameter = 10 m

$\Rightarrow$ Radius (R) = $\frac{10}{2}$ m = 5 × 100 cm = 500 cm

Height (H) = 2 m = 2 × 100 cm = 200 cm

$\Rightarrow$ Volume of the cylindrical tank

$\pi R^2 H = \pi \times (500)^2 \times 200\ cm^3$

Now, time required to fill the tank

$$= \frac{[\text{Volume of the tank}]}{[\text{Volume of water flown in 1 hour}]} = \frac{\pi \times 500 \times 500 \times 200}{\pi \times 30000000} hrs$$

$$= \frac{5 \times 5 \times 2}{30} hrs = \frac{5}{3} hrs = \frac{5}{3} \times 60 \text{ minutes} = 100 \text{ minutes}.$$